MULTIMEDIA UNIVERSITY

STUDENT ID NO

MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 2, 2018/2019

BSP2014 – INTRODUCTION TO APPLIED PROBABILITY AND STOCHASTIC PROCESSES

(All sections / Groups)

4 MARCH 2019
9 a.m. – 11 a.m.
(2 Hours)

INSTRUCTIONS TO STUDENTS

1. This question paper consists of **FIVE (5)** printed pages inclusive of the cover page, formulae sheet.
2. Answer **ALL** four questions in the answer booklet provided.
3. Students are allowed to use authorized calculators by lecturer only.
4. Marks are shown at the end of each question.

Question 1 [20 marks]

(a) Recent statistics suggest that 25% of those who visit Uniqlo website make a purchase. The retailer would like to verify the claim. To do so, she selected a sample of 8 "hits" to her site.

i) Find the probability that less than 2 "hits" resulted in a purchase. [5 marks]

ii) Find the probability that 5 of the "hits" will end up without any purchase. [3 marks]

iii) Find the standard deviation on the number of "hits" with a purchase. [2 marks]

(b) Waiting times to received food after placing an order at the local Subway sandwich shop is on average 60 seconds.

i) Calculate the probability that a customer waits more than 0.6 minutes. [5 marks]

ii) Calculate the probability that a customer waits between 40 and 70 seconds. [5 marks]

Question 2 [20 marks]

(a) Given that cars passing KLCC tower with the rate of 3 per second starting from 7am,

i) Find the probability that 6 cars passing KLCC tower for the first 5 seconds. [4 marks]

ii) Find the probability that time between 2 cars passing the KLCC tower is less than 1 second. [3 marks]

iii) Find the probability that time between 2 cars passing the KLCC tower is between 0.5 seconds and 1.5 seconds. [6 marks]

(b) A random walk starts from the origin. The chances for the walker to move to the right are 4 times the chances he moves to the left.

i) Find the probability that the walker is at the position 2 after 8 steps. [4 marks]

ii) Find the probability that the walker is at position -3 after 3 steps. [3 marks]

Continued...

Question 3 [30 marks]

(a) Given a continuous random variable X has the following cumulative density function:

$$F(x) = 3x^2 - 2x^3 \quad \text{for} \quad 0 < x \le 1$$

i) Find the probability density function. [3 marks]
ii) Find the moment generating function. [9 marks]

(b) Random variables X and Y have the following function:

$$f(x,y) = \begin{cases} \dfrac{kx^2}{y} & ; \ x = 1,2 \ ; \ y = 2,3,4 \\ 0 & ; \ otherwise \end{cases}$$

i) Find the value k. [3 marks]
ii) Determine the marginal pmf for X and Y. [8 marks]
iii) Find P $(X + Y \le 4)$. [3 marks]
iv) Determine whether X and Y independent. Explain. [4 marks]

Question 4 [30 marks]

Consider a Markov chain with state space of {1 , 2 , 3} and transition probability matrix as below.

$$\begin{bmatrix} 0 & 0.4 & 0.6 \\ 0.25 & 0.75 & 0 \\ 0.4 & 0 & 0.6 \end{bmatrix}$$

(a) Draw the transition diagram. [3 marks]
(b) Find $P(X_2 = 1 \mid X_0 = 2)$. [7 marks]
(c) Determine whether state 2 is transient or persistence state. Find the period of state 2. [10 marks]
(d) Determine whether each of the states to be absorbing or non-absorbing state. [6 marks]
(d) Let the initial distribution to be 0.4, 0.2 and 0.4. Find the probability of state 2 after 2 steps. [4 marks]

End of Page

FORMULAE

A. PROBABILITY DISTRIBUTION

Bernoulli Probability Distribution $P(X=x)=p^x q^{1-x}$	for $x=0, 1$
Binomial Probability Distribution $P(X=x)={}^nC_x\, p^x q^{n-x}$	for $x=0, 1, \ldots, n$
Poisson Probability Distribution $P(X=x)=\dfrac{e^{-\lambda}\lambda^x}{x!}$	for $x=0, 1, 2, \ldots$
Geometric Probability Distribution $P(X=x)=pq^{x-1}$	for $x=1, 2, \ldots$
Uniform Probability Distribution $f(x)=\dfrac{1}{b-a}$	for $a<x<b$
Exponential Probability Distribution $f(x)=\lambda e^{-\lambda x}$	for $x>0$

B. MOMENT

k^{th} Moment about the origin of X: $\mu'_k = E[X^k]$
k^{th} Moment about the mean of X: $\mu_k = E[X-\mu]^k$
Moment Generating Function for X: $M_X(t)=E\left(e^{tx}\right)$
Derivative of Moment Generating Function for X: $M_X^{(r)}(t)=E\left(X^r e^{tx}\right)$

C. STOCHASTIC PROCESS

Simple Random Walk:

$$P(X_n = m) = \binom{n}{\frac{n+m}{2}} p^{\left(\frac{n+m}{2}\right)} q^{\left(\frac{n-m}{2}\right)}$$ where m and n are both even or both odd.

$E(X_n) = n(p - q)$; $Var(X_n) = 4npq$

Gambler's Ruin Problem

$$P_a = \begin{cases} \dfrac{\left(\frac{q}{p}\right)^a - \left(\frac{q}{p}\right)^c}{1-\left(\frac{q}{p}\right)^c} & , \quad p \neq q \\ 1-\dfrac{a}{c} & , \quad p = q \end{cases} \quad ; \quad P_a + P_b = 1$$

$$D_a = \begin{cases} \dfrac{1}{q-p}\left\{a - c\left[\dfrac{1-\left(\frac{q}{p}\right)^a}{1-\left(\frac{q}{p}\right)^c}\right]\right\} & , \quad p \neq q \\ a(c-a) & , \quad p = q \end{cases}$$

D. POISSON PROCESS

Pmf for N(t) :

$$P\{N(t) = n\} = \frac{e^{-\lambda t}(\lambda t)^n}{n!}$$

Pdf for T_n:

$$f(t) = \lambda e^{-\lambda t}$$

Pdf for S_n:

$$f(t) = \frac{\lambda e^{-\lambda t}(\lambda t)^{n-1}}{(n-1)!}$$